The Rolling Ball Viscometer

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A study of the system of the inclined tube and rolling ball as applied to the measurement of viscosity is described. Dimensional analysis was used to derive general relations between the variables involved and the simple calibration for the rolling ball viscometer in the streamline region of fluid flow. The coefficient of the calibration equation may be calculated from the dimensions of the instrument with the aid of an experimentally determined empirical factor. By using the equations given, the useful range of the rolling ball viscometer may be predicted without experimental calibration or an instrument may be designed for measurements over any desired range of viscosity. An empirical correlation is given which allows viscosity to be estimated from data taken on the viscometer in the turbulent region of fluid flow. The effect of temperature changes on the viscometer and its calibration is discussed.

FOR many years the system of the inclined tube and rolling ball has been used as an empirical instrument for viscosity measurement without complete knowledge of the general relations existing between the variables involved. The instrument has been used because it is more easily adaptable for measurements in enclosed systems. It has many advantages, which may be listed as follows:

1. The apparatus can be extremely simple.
2. Only a small sample of material is required.
3. Visual observation in glass apparatus is possible even with opaque liquids, since the ball is in contact with the tube at one point.
4. The system possesses great flexibility, with the opportunity of changing one or more of the variables: tube diameter, ball diameter, angle of inclination, ball density, and rolling distance of ball.

The use of the system of the inclined tube and rolling ball as a viscometer was first suggested by Flowers (*7*) and was studied by Hersey (*8*), who evolved by dimensional treatment the manner of correlation of the variables involved. A calibration first used by Hersey and Shore (*9*) consisted of a plot of the equation

$$\beta \frac{\mu}{\rho\sqrt{\frac{\rho_s}{\rho} - 1}} = z\sqrt{\frac{\rho_s}{\rho} - 1} \qquad (1)$$

Except at high rolling velocities this relation was linear, and the line extended passed through the origin.

Sage (*13*) described the use of the system in measuring the viscosity of hydrocarbon solutions. He used a calibration of the form

$$\mu = b\,z\,(\rho_s - \rho) \qquad (2)$$

which may readily be derived from Equation 1. This relation also departed from the linear function through the origin, but only for viscous fluids and low rolling velocities.

Sage and Lacey (*14*) measured the viscosity of hydrocarbon gases in a similar apparatus and worked in the turbulent region of flow to a large extent. The value of constant b in Equation 2, which represented the calibration in the streamline region only, was obtained from observations on known liquids. Using other fluids for the turbulent region, but still calculating viscosity by Equation 2, a viscosity-ratio correction factor obtained for each fluid was plotted against a function proportional to Reynolds number. By use of this plot applicable to the one instrument only, the viscosity of fluids flowing with turbulence was calculated by the method of successive trials.

Block (*5*) has recently suggested the addition of a term containing an empirically derived exponent to the calibration Equation 1 to effect agreement of the equation with experimental results in the turbulent region.

Hoeppler (*10*) reported the results of experimental work on the eccentric fall of large spheres in a tube inclined at an angle of 80°. In suggesting that this arrangement of the inclined tube and rolling ball be used as a viscometer, he too employed Equation 2 as a calibration. The commercial instrument bearing his name uses a short, nearly vertical glass tube of large diameter (16 mm.) and close fitting balls of either glass or steel.

No general study of the system of the inclined tube and rolling ball has been reported. An experimental investigation of the system was therefore made on tubes from 6 to 10 mm. in diameter with balls of aluminum, steel, and brass ranging in size from 85 per cent to the full tube diameter. The general correlation obtained verified the viscometer calibration, Equation 2, and in addition indicated a method by which the unknown coefficient, b, could be calculated from the dimensions of the apparatus with the aid of an empirical correlation.

Nomenclature

b, c = proportionality constants
C = coefficient, defined by Equation 15
d = diameter of ball, cm.
D = diameter of tube, cm.
f = resistance factor = $R/(h^2\,\rho\,u^2)$, dimensionless
f_c = resistance factor at critical velocity
F = force, gram cm. per second^2
g = acceleration of gravity = 980 cm. per second^2
h = equivalent diameter of annular space between ball and tube, defined by Equation 7, cm.
K = correlation factor, dimensionless
L = length, a fundamental dimension, cm.
M = mass, a fundamental dimension, grams
R = driving force on ball or resistance of fluid to motion of ball, defined by Equation 8, gram. cm. per second^2
Re = Reynolds number = $(h\,u\,\rho)/\mu$, dimensionless
Re_c = Reynolds number at critical velocity
t = temperature, ° C.
T = time, a fundamental dimension, seconds
u = average fluid velocity through annular space between ball and tube, cm. per second
V = terminal rolling velocity of ball, cm. per second
z = time of roll, seconds
α_b = linear coefficient of thermal expansion of ball material, per ° C.
α_t = coefficient of expansion of tube, per ° C.
β = coefficient in Equation 1
δ = prefix indicating derivative
Δ = increment of change of variable
θ = angle of inclination of tube to the horizontal
μ = viscosity of fluid, grams per cm. per second
μ_0 = viscosity of fluid calculated when Equation 14 is used in the turbulent region, gram per cm. per second
π = 3.1416
ρ = density of fluid, grams per cc.
ρ_s = density of ball, grams per cc.
ϕ = symbol for "is function of"
o = subscript denoting value at temperature of calibration of viscometer

Dimensional Analysis

There are seven variables to be considered in the analysis of the rolling ball viscometer. In addition to the fundamental units—length, L; mass, M; and time, T—force, $F = MLT^{-2}$, is considered a unit of its own kind. This can

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be done because the system is in equilibrium and in unaccelerated motion, and no use is made of the fact that where there happens to be accelerated motion, force is equal to mass times acceleration. The seven variables with symbols and dimensions are:

Variable	Symbol	Dimension
Diameter of tube	D	L
Diameter of ball	d	L
Velocity of motion	V	LT^{-1}
Density of ball	ρ_s	ML^{-3}
Density of fluid	ρ	ML^{-3}
Viscosity of fluid	μ	$FL^{-2}T$
Acceleration of gravity	g	FM^{-1}

The dimensional formula of viscosity is obtained directly from its definition of force per unit area per unit velocity gradient. The intensity of gravity is taken with the dimensions FM^{-1} because the equations of motion in this case will not use the accelerating aspect of gravitational motion but only the intensity of the force exerted by gravity upon unit mass. Because of the inclination of the tube at the angle θ to the horizontal, the effective acceleration of gravity is g sine θ.

There are seven variables and four kinds of units; therefore three dimensionless products or groups of variables must be found. Two of these groups, the ratios d/D and ρ_s/ρ, may be written immediately by inspection. The third dimensionless product, obtained by the method of Bridgman (*6*), includes five variables in the form $V^{-1}\ \mu^{-1}\ d^2\ \rho\ g$ sine θ. The final general relation is

$$\phi\ (V^{-1}\ \mu^{-1}\ d^2\ \rho\ g \text{ sine } \theta)\ \phi'\left(\frac{\rho_s}{\rho}\right)\ \phi''\left(\frac{d}{D}\right) = 0 \tag{3}$$

The velocity of the ball rolling down an inclined tube is given by the equation

$$V = c\,\frac{d^2\ \rho\ g \text{ sine } \theta}{\mu}\ \phi'\left(\frac{\rho_s}{\rho}\right)\ \phi''\left(\frac{d}{D}\right) \tag{4}$$

The viscosity of the fluid is expressed as a function of all the variables by the relation

$$\mu = c\,\frac{d^2\ \rho\ g \text{ sine } \theta}{V}\ \phi'\left(\frac{\rho_s}{\rho}\right)\ \phi''\left(\frac{d}{D}\right) \tag{5}$$

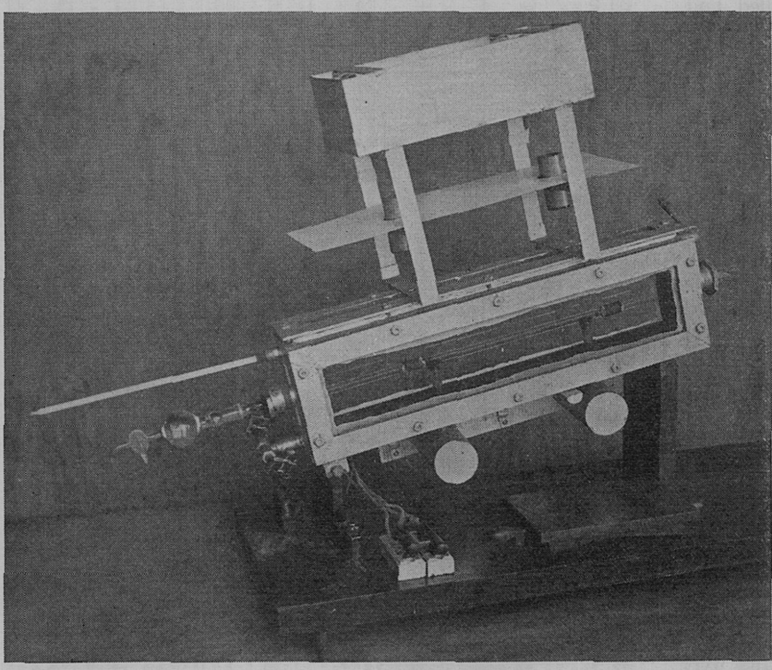

FIGURE 1. EXPERIMENTAL APPARATUS, ROLLING BALL IN INCLINED TUBE

The solution of the entire problem is obtained if the unknown functions are evaluated.

The resistance to the motion of the ball is developed by the fluid in being accelerated and decelerated in passing through the constriction between the ball and tube. The average fluid velocity through this space is related to the ball velocity by the equation

$$\frac{u}{V} = \frac{d^2}{D^2 - d^2} \tag{6}$$

The linear dimension commonly employed in hydraulics for noncircular channels is the equivalent diameter, equal to four times the hydraulic radius, which is defined as the cross-sectional area of the channel divided by the wetted perimeter. In this system the equivalent diameter of the crescent-shaped space is theoretically

$$h = 4\,\frac{\pi}{4}\,\frac{D^2 - d^2}{\pi\ (D + d)} = D - d \tag{7}$$

The dimensional analysis can be simplified by combining the factors d, ρ_s, ρ, and θ into a force term, the driving force on the ball, equal to the resisting force of the fluid since the ball rolls with unaccelerated motion. This term is represented by the equation

$$R = \frac{5}{7}\,g \text{ sine } \theta\,\frac{\pi d^3}{6}\,(\rho_s - \rho) \tag{8}$$

The coefficient 5/7 is that fraction of the effective force of gravity that causes translational motion of the rolling sphere.

In the dimensional treatment of a similar problem, Awberry and Griffiths (*1*) included Reynolds number as one of the dimensionless products. In a second application of dimensional analysis to the present system, the variables used are:

Variable	Symbol	Dimension
Driving force	R	MLT^{-2}
Equivalent diameter	h	L
Density of fluid	ρ	ML^{-3}
Viscosity of fluid	μ	$ML^{-1}T^{-1}$
Velocity of fluid	u	LT^{-1}

There are now five variables and three kinds of fundamental units; therefore two dimensionless products must be found. If the Reynolds number, $(h\ u\ \rho)/\mu$, is assumed to be one dimensionless product, the other is shown to be $R/(h^2\ \rho\ u^2)$, which group will now be called the *resistance factor*. A general relation having only one unknown function and whose terms are capable of evaluation by experiment is written

$$\frac{R}{h^2\ \rho\ u^2} = \phi\,\frac{h\ u\ \rho}{\mu} \tag{9}$$

Apparatus and Experimental Methods

In order to evaluate the functions of Equations 5 and 9, experimental equipment designed to permit variation of all factors was constructed.

The apparatus consisted of a precision-bore glass tube (procured from the Fish-Schurman Corp., New York, N. Y., and also available from the Fischer and Porter Co., Hatboro, Penna.) in an isothermal water bath and an automatic photoelectric device for recording the time required for a rolling ball to traverse a known distance in the tube. A photograph of a tube in its bath is shown in Figure 1.

TABLE I. SUMMARY OF EXPERIMENTAL CONDITIONS AND RESULTS

Tube	Tube Diameter	Ball Diameter	Diameter Ratio	No. of Experimental Points	Range of Reynolds No.	Critical Reynolds No.	Correlation Factor, K Calculated	Correlation Factor, K Graphical
	Cm.	Cm.						
I	0.5994	0.5951	0.9928	38	0.31–18.0	..	6.22×10^{-7}	....
		0.5937	0.9905	45	0.43–32.6	..	1.16×10^{-6}	....
		0.5861	0.9778	37	1.8–135	21.5	7.13×10^{-6}	....
		0.5785	0.9650	38	5.1–236	18.0	....	2.00×10^{-5}
		0.5709	0.9523	21	35–330	(15.5)	....	4.00×10^{-5}
		0.5556	0.9269	107	8.2–511	13.0	....	1.06×10^{-4}
		0.5144	0.8581	37	32–505	(9.8)	....	4.17×10^{-4}
II	0.6485	0.5951	0.9177	33	20–270	(12.0)	....	1.54×10^{-4}
		0.5785	0.8921	33	30–344	(11.0)	....	2.78×10^{-4}
		0.5556	0.8568	99	15–448	(10.0)	....	4.60×10^{-4}
III	0.8014	0.7950	0.9921	23	0.97–13.1	..	6.84×10^{-7}	....
		0.7938	0.9905	61	1.03–65	..	1.02×10^{-6}	....
		0.7525	0.9390	62	0.036–705	13.0	7.36×10^{-5}	....
		0.7144	0.8914	237	0.0046–806	10.6	2.62×10^{-4}	....
IV	0.9997	0.9906	0.9909	51	1.5–122	35.0	1.13×10^{-6}	....
		0.9830	0.9832	20	18–263	24.0	....	3.94×10^{-6}
		0.9754	0.9756	40	12–406	20.0	....	7.15×10^{-9}
		0.9677	0.9680	21	63–537	(18.5)	....	1.59×10^{-5}
		0.9525	0.9527	109	13–916	17.5	....	4.65×10^{-5}
		0.9112	0.9115	31	60–936	(11.5)	....	1.67×10^{-4}
		0.8731	0.8734	44	38–383	(10.0)	....	3.45×10^{-4}

Values in parentheses obtained by extrapolation.

Light from two lamps was focused onto the upper surface near the ends of the glass tube. Light passing through the tube was conducted through quartz tubes to two gas-filled photoelectric cells. These cells were part of circuits which, through sensitive relays, controlled an electric chronoscope. When the light to the first photocell was interrupted by a ball rolling down the inclined tube, time measurement by the chronoscope was started. Similarly, the ball stopped the time measurement in passing through the second light beam.

The inclination of the tube and the distance traversed by the balls were measured with a cathetometer. The roll distance between the light beams was measured as the distance between the positions of stationary balls placed in each light beam at the point at which they just caused the relays to operate the chronoscope. The inclination of the tube was varied from 4° to 25° from the horizontal. The distance between the light beams was about 17 cm.

The glass tube was carefully cleaned before use. Water maintained at a constant temperature was circulated through the jacket by a pump. After the water jacket attained the desired temperature, the tube and reservoir at its lower end were filled with the liquid used. At each inclination about ten balls of each size and material were introduced into the open end and successively rolled down the tube. The balls were removed from the receiver at the lower end of the tube, the tube was refilled, and the procedure was repeated at another inclination. The roll velocity was calculated from the known distance and the average of the values of roll time.

Four Jena KPG glass tubes were used in this work. Their inside diameters were measured with plug gages. Seventeen different ball sizes were used. Steel balls are manufactured to a high degree of precision and were assumed to be their nominal diameter. Close fitting aluminum balls were specially measured by the manufacturer (Hoover Ball and Bearing Co., Ann Arbor, Mich.). Table I gives the diameter of the tubes and balls used.

Sixteen fluids were used. Except for air, water, ethyl alcohol, and solutions of ethanol and sucrose, which are accepted as standards for viscometer calibration, the viscosity of the fluids was measured in Bingham (*3*) or Ostwald capillary tube viscometers. The density of all fluids was measured experimentally. The viscosity of the fluids varied from about 0.23 to 144 centipoises, the density from 0.62 to 1.61 grams per cc. The measurements were made to an estimated precision as follows:

Tube diameter	0.08 per cent
Ball diameter	0.01 to 0.05 per cent
Ball density	0.03 per cent
Fluid density	0.02 per cent
Roll distance	0.1 per cent
Roll time	0.2 per cent
Inclination of tube	0.05 to 0.1 per cent

The major source of error was the change in viscosity of the fluid with slight changes in temperature, caused by contact of the fluid with the rolling balls. Most of the work was conducted at room temperature. At other temperatures the balls were brought to operating temperature in a separate container jacketed with the circulating water. An over-all precision in calculated rolling velocity from 0.5 to 1 per cent was obtained on measurements made under similar conditions at different times.

Experimental Results

The conditions used in the experimental work more than covered the useful range for viscosity measurement. The streamline region of fluid flow, which is the region having characteristics suitable for viscosity measurement, was covered over its most useful range. The turbulent region of flow, often used but not as advantageous for viscosity measurement, was also covered.

The turbulent region was characterized by a deviation from the relations existing in the streamline region of flow. The deviation is probably due to the inertia of the fluid, to the formation of eddy currents in the flowing fluid, or to a combination of these causes. Block (*5*) has expressed the opinion that inertia and not turbulence accounts for the deviation. No attempt was made in this work to determine the causes and only an empirical correlation was obtained.

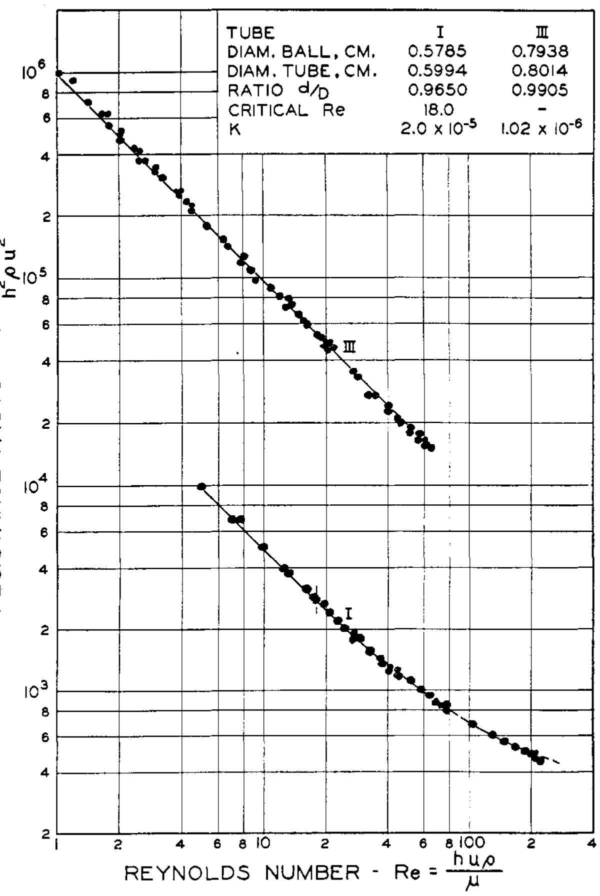

FIGURE 2. TYPICAL CORRELATION FOR SYSTEM ROLLING BALL IN INCLINED TUBE

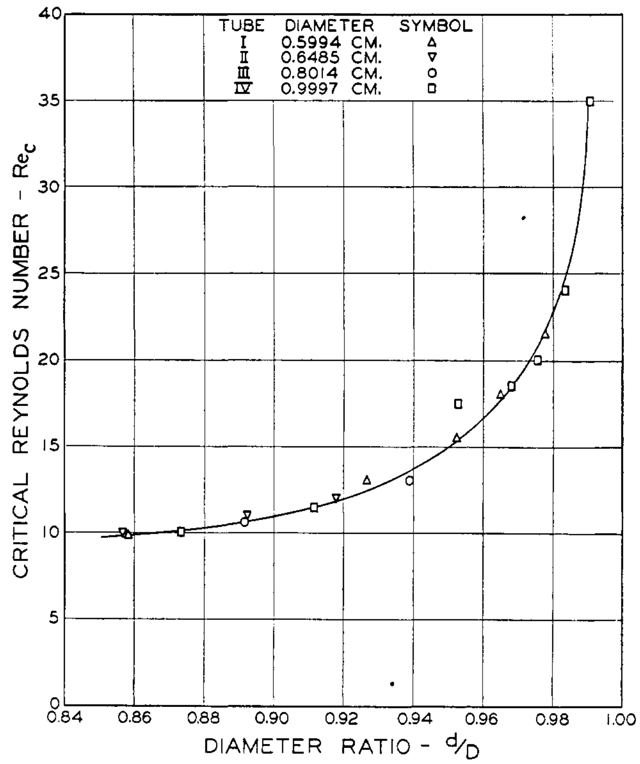

FIGURE 3. CRITICAL REYNOLDS NUMBER FOR ROLLING BALL VISCOMETER

The observed motion of the ball was fundamentally a steady rolling motion at constant velocity. Block (*4*) observed and measured the extent of sliding motion in combination with rolling in more viscous fluids at angles above 13°. Sliding was observed in this work with viscous oils at higher inclinations. With the least viscous fluids sliding was not apparent. The critical velocity at which the fluid flow changed from streamline to turbulent could not be determined by direct observation of the rolling balls. To a large extent the motion of the ball in both the streamline and turbulent regions was uniform and the data were reproducible. The limit of usefulness of the viscometer was reached before the motion of the ball became visibly irregular.

The experimental data were grouped according to ball and tube size or ratio of diameter of ball to diameter of tube, d/D. For each combination the values of Reynolds number and the resistance factor were calculated and plotted on logarithmic coordinate paper. The twenty-one plots obtained were similar in every respect to the fluid friction plot for flow through pipe. Two representative curves are shown in Figure 2.

In the region of streamline flow the experimental data fell on a straight line of slope −1 (upper curve of Figure 2). The turbulent region was represented by a smooth concave curve as shown by the lower curve. Within the limits of the experimental data all curves had the same shape. Regardless of the values of ball and tube diameter, the location of the curve on the coordinate system was dependent only on the ratio d/D.

For streamline flow the equation of the straight line through the plotted data was expressed as

$$\log \frac{R}{h^2 \rho u^2} = -\log \frac{h u \rho}{\mu} + \log \frac{1}{K} \tag{10}$$

which became Equation 11.

$$\frac{R}{h^2 \rho u^2} = \frac{1}{K} \frac{\mu}{h u \rho} \tag{11}$$

After substituting the values of the equivalent diameter, h, and the driving force, R, given by Equations 7 and 8, respectively, Equation 11 was written

$$\mu = \frac{5\pi}{42} K \frac{d^2 \rho g \text{ sine } \theta}{u} \frac{\rho_s - \rho}{\rho} \frac{d}{D - d} \tag{12}$$

By expressing the fluid velocity in terms of the ball velocity, V, the above relation became

$$\mu = \frac{5\pi}{42} K \frac{d^2 \rho g \text{ sine } \theta}{V} \frac{\rho_s - \rho}{\rho} \frac{D + d}{d} \tag{13}$$

Equation 13 is dimensionally correct and similar to Equation 5. The correlation factor, K, must be included with the term $(D + d)/d$ as a part of the function ϕ'', since it is shown to be a function of the ratio d/D.

When D, d, θ, and K are constant, Equation 13 reduces to

$$\mu = C \frac{\rho_s - \rho}{V} \tag{14}$$

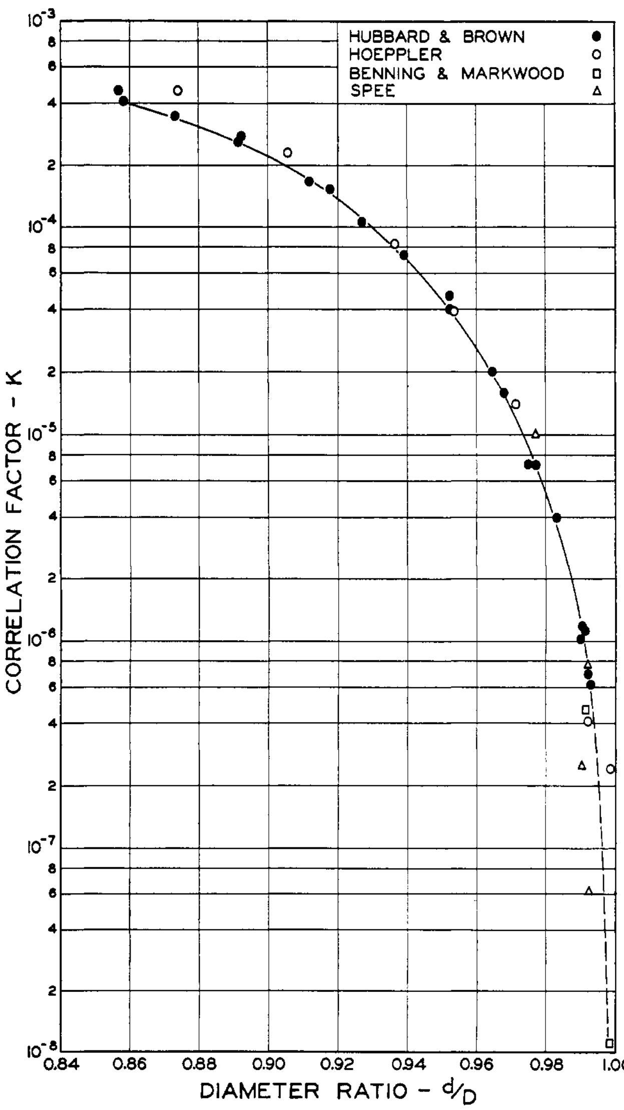

FIGURE 4. CORRELATION FOR ROLLING BALL VISCOMETER

Table II. Calculations from Data of Hoeppler (10)

Fluid	$\left(\frac{\text{Radius of Tube}}{\text{Radius of Ball}}\right)^2$	Diameter Ratio, d/D	Ball Diameter, d	Driving Force, R	Reciprocal Velocity	Roll Velocity, V	Fluid Velocity, u	Resistance Factor, f	Reynolds Number, Re	Correlation Factor, K
		Cm.	Cm.	Gm. cm./sec.^{2}	Sec./cm.	Cm./sec.	Cm./sec.			
Air[a]	1.002	0.9990	1.5984	11.46×10^3	.....	0.2	100.2	3.84×10^8	1.07	2.4×10^{-7}
Castor oil[b]	1.016	0.9921	1.5874	9.844×10^3	1810	0.000552	0.0345	5.39×10^{10}	4.54×10^{-5}	4.1×10^{-7}
	1.06	0.9713	1.5541	9.235	54.7	0.01828	0.305	4.90×10^7	1.46×10^{-3}	1.4×10^{-5}
	1.10	0.9535	1.5256	8.736	20.01	0.0500	0.500	6.60×10^6	3.87×10^{-3}	3.9×10^{-5}
	1.14	0.9366	1.4986	8.280	9.92	0.1008	0.720	1.62×10^6	7.50×10^{-3}	8.3×10^{-5}
	1.22	0.9054	1.4486	7.479	3.77	0.2653	1.205	2.34×10^5	1.90×10^{-2}	2.3×10^{-4}
	1.31	0.8737	1.3979	6.722	1.96	0.5102	1.644	6.34×10^4	3.46×10^{-2}	4.6×10^{-4}

[a] Tube: diameter = 1.600 cm. assumed; inclination = 80°; sine θ = 0.9848. Ball: steel; ρ_s = 7.77. Fluid: air at 20.0° C.; $\mu = 1.81 \times 10^{-4}$; ρ = 0.0012; $\rho_s - \rho$ = 7.769.

[b] Tube: diameter = 1.600 cm. assumed; inclination = 80°; sine θ = 0.9848. Ball: steel; ρ_s = 7.775. Fluid: castor oil at 20.0° C.; μ = 9.204; ρ = 0.960 (assumed); $\rho_s - \rho$ = 6.815.

which is equivalent to Equation 2, most generally used to calculate viscosity from experimentally determined values of roll time or velocity.

Values of K were calculated from the experimental data by the method of least squares for 8 out of the 21 cases. For the remaining cases K was determined by inspection of the data. Since all the curves had the same shape, they could be superimposed by transposition. An aid to this process was a template made in the form of an average curve in the turbulent region and placed in proper relation to the straight line in the streamline region with a definite critical Reynolds number indicated at the intersection of the line and the curve. This template was superimposed on each of the plots and fitted to the experimental data in the most favorable manner. Values of the critical Reynolds number, Re_c, and the corresponding critical resistance factor, f_c, were read from the plot. The curve drawn through the data of the lower curve in Figure 2 shows the position of the template on this plot.

Values of the correlation factor, K, and the critical Reynolds number, Re_c, are given in Table I for each ball and tube combination. Most weight must be given to the calculated values of K, but many of the graphical values are based on enough points in the streamline region to assume almost equal weight. Figure 3 shows the critical Reynolds number and Figure 4 the correlation factor plotted as functions of the diameter ratio.

The value of constant C of Equation 14 can be calculated from the dimensions of the apparatus and the corresponding value of K from Figure 4 by the relation

$$C = \frac{5\pi}{42} K \, g \text{ sine } \theta \, d(D + d) \tag{15}$$

For experimental determination of viscosity, however, it is recommended that the value of C be determined by experiment with fluids of known viscosity and density.

Equation 14 is the valid calibration for the rolling ball viscometer only in the streamline region of fluid flow. This region is defined as having a Reynolds number, $(h\ u\ \rho)/\mu$, smaller than the critical value given in Figure 3. In practice the condition of flow is checked by showing that the resistance factor is greater than the corresponding critical resistance factor. The opposite limit of the streamline region was estimated to have a resistance factor of about ten million.

For any resistance factor smaller than the value corresponding to the critical Reynolds number, there are values of Reynolds number $(h\ u\ \rho)/\mu$ and $(h\ u\ \rho)/\mu_0$, which respectively represent the abscissas of the curve and the straight line of Equation 10 extended beyond the critical Reynolds number. A ratio of these values is equal to μ/μ_0, which is the ratio of the true viscosity to that calculated from the calibration of the instrument in the streamline region. For each experimental point known to lie in the turbulent region, this ratio and the ratio of the resistance factor to the critical resistance factor were calculated. The plotted points with a curve drawn through them are shown in Figure 5. The scattering of the points is not unusual in this transition zone between the regions of streamline and turbulent flow.

The curve of Figure 5 is drawn through points obtained for relatively close fitting balls and is a general correlation from which viscosity may be calculated from data taken with the rolling ball viscometer in the turbulent region of flow. The diameter ratio will be known. The viscometer will have been calibrated in the streamline region with known fluids, or the value of the correlation factor may be read from Figure 4. The value of the critical Reynolds number may be read from Figure 3. The corresponding critical resistance factor is then calculated by substituting these values in Equation 11. Corresponding values of the resistance factor, f, and the Reynolds number, Re, may be calculated from the experimental roll velocity and the known driving force by means of Equation 11. If the resistance factor, f, is smaller than the critical resistance factor, f_c, the data were taken in the turbulent region of flow. Nevertheless the viscosity, μ_0, is calculated from Equation 14. Corresponding to the ratio of the resistance factors f/f_c, the ratio of the true viscosity to the viscosity just calculated is read from Figure 5.

The experimental work done in the turbulent region was very extensive and reached the point at which the rolling motion of the ball ceased to be uniform. The absolute limit of applicability of the instrument as a viscometer is a Reynolds number of about 800. Since the instrument loses its sensitivity in the turbulent region, its use is not recommended when ratio f/f_c is less than 0.25.

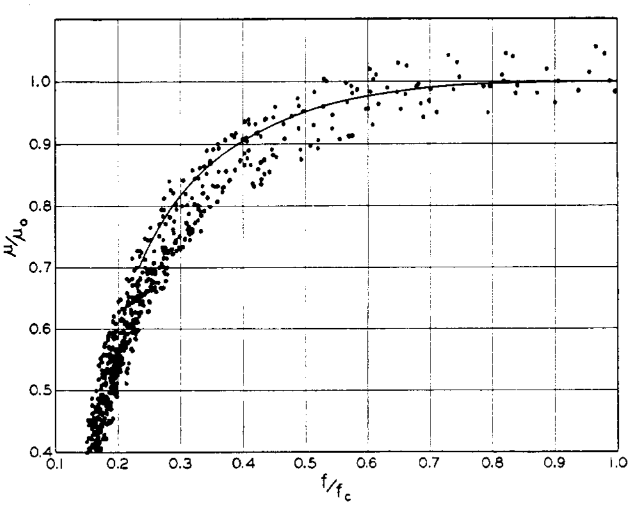

Figure 5. Correction for Rolling Ball Viscometer in Region of Turbulent Flow

TABLE III. CALCULATIONS FROM DATA OF BENNING AND MARKWOOD (2) AND SPÉE (16)

Fluid	Tube Diameter, D Cm.	Ball Diameter, d Cm.	Diameter Ratio, d/D Cm.	Temp. ° C.	Fluid Viscosity, μ G./cm. sec.	Ball Density, ρ_s G./cc.	Fluid Density, ρ G./cc.	$\rho_s - \rho$ G./cc.	Driving Force, R G. cm./sec.[2]	Roll Time Sec.	Roll Velocity V Cm./sec.	Fluid Velocity, u Cm./sec.	Resistance Factor, f	Reynolds Number, Re	Correlation Factor, K
Air[a]	1.5934	1.5905	0.9982	1.4	1.715×10^{-4}	2.405	0.0013	2.404	3.492×10^{3}	36.0	0.278	76.0	5.59×10^{7}	1.652	1.084×10^{-8}
				1.7	1.717	2.405	0.0013	2.404	3.492	36.1	0.277	75.8	5.60	1.643	1.087
				30.7	1.859	2.404	0.0012	2.403	3.491	38.3	0.261	71.5	6.98	1.298	1.105
				40.0	1.904	2.403	0.0011	2.402	3.489	39.0	0.256	70.2	7.49	1.205	1.110
				49.7	1.949	2.403	0.0011	2.402	3.489	39.6	0.253	69.2	7.93	1.125	1.121
				79.5	2.086	2.402	0.0010	2.401	3.488	41.6	0.240	65.8	9.59	0.915	1.140
Water[a]	1.5934	1.5805	0.9919	0.5	17.63×10^{-3}	2.403	1.000	1.403	2.00×10^{3}	144.9	0.0690	4.21	67.8×10^{4}	3.08	4.79×10^{-7}
				30.0	8.00	2.402	0.996	1.406	2.005	65.4	0.1530	9.34	13.8	15.00	4.81
				59.6	4.73	2.401	0.983	1.418	2.020	38.5	0.260	15.85	4.92	42.5	4.79
Chloroform[a]	1.5934	1.5805	0.9919	29.67	5.12×10^{-3}	2.402	1.4079	0.9311	1.327×10^{3}	63.4	0.1578	9.61	5.88×10^{4}	35.6	4.78×10^{-7}
				0.46	6.97	2.4026	1.5256	0.8770	1.250	91.1	0.1096	6.69	11.00	18.9	4.81
				29.67	5.12	2.4017	1.4709	0.9308	1.328	63.4	0.1578	9.61	5.88	35.6	4.78
				59.36	3.92	2.4008	1.4142	0.9866	1.406	46.0	0.2174	13.25	3.37	61.6	4.81
				0.46	6.97	2.4026	1.5256	0.8770	1.250	95.1	0.1052	6.41	11.98	18.1	4.61
				30.91	5.07	2.4017	1.4683	0.9334	1.330	64.6	0.1548	9.44	6.11	35.2	4.64
				29.93	5.12	2.4013	1.4702	0.9315	1.328	65.2	0.1534	9.36	6.18	34.7	4.66
Freon-113[a]	1.5934	1.5805	0.9919	0.48	9.28×10^{-3}	2.4026	1.6202	0.7824	1.228×10^{3}	141.5	0.0707	4.32	24.35×10^{4}	9.75	4.21×10^{-7}
				29.88	6.19	2.4017	1.5534	0.8483	1.221	86.9	0.1150	7.02	9.58	22.8	4.58
				0.47	9.28	2.4026	1.6202	0.7824	1.228	141.2	0.0708	4.32	24.35	9.75	4.21
Air[b]	2.012	1.997	0.9925	19.5	1.813×10^{-4}	1.174	0.001	1.173	4.379×10^{3}	...	....	99.1	1.65×10^{6}	9.85	6.2×10^{-8}
	1.281	1.271	0.9922	19.8	1.815	1.1646	0.0012	1.1634	1.122	...	....	471.	4.22×10^{4}	31.0	7.6×10^{-7}
	2.017	1.997	0.9901	19.5	1.813	1.174	0.001	1.173	4.379	...	....	294.3	1.09×10^{5}	37.0	2.5×10^{-7}
Water[b]	1.301	1.271	0.9769	12.0	12.78×10^{-3}	1.1646	0.9995	0.1651	159.0	...	....	4.151	1.03×10^{4}	9.75	1.0×10^{-5}

[a] From Benning and Markwood: tube inclination = 80°; sine θ = 0.9848; roll distance = 10.00 cm.
[b] From Spée: tube inclination = 66°; sine θ = 0.9136.

Effect of Temperature Changes

The effect of temperature changes on the rolling ball viscometer is appreciable when high temperatures are employed or when materials with high coefficients of thermal expansion are used. The effect is greater if the ball and tube are of different material and when the ratio of diameters approaches unity.

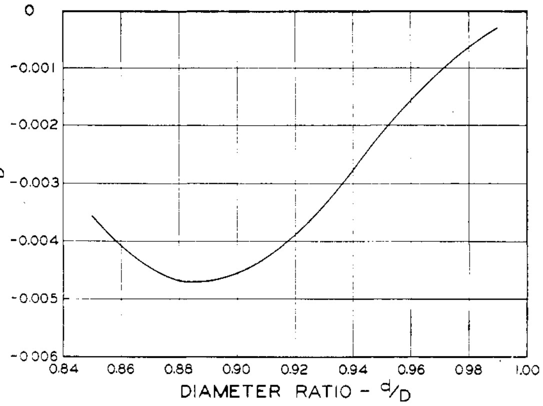

FIGURE 6. RATE OF CHANGE OF CORRELATION FACTOR WITH DIAMETER RATIO

When the rolling ball viscometer is used at a temperature other than that at which it was calibrated, the density of the fluid at the temperature used must be known. The value of factor C of Equation 14 will change as the operating temperature is changed, and the direction and magnitude of the change may be calculated from a knowledge of the average linear coefficients of expansion of the tube and ball materials. The general calibrating equation is written in the form

$$\mu = \frac{5\pi}{42} g \text{ sine } \theta \frac{\rho_s - \rho}{V} K \, d(D + d) \qquad (16)$$

Temperature changes have an effect only on the terms ρ_s, K, and $d(D + d)$. The coefficient of the calibrating equation must be estimated at each temperature and is the product of the numerical factor and the variable terms of the above equation.

EFFECT ON TUBE. The increase in length and diameter of the tube is calculated from the dimensions and average linear coefficient of expansion of the tube material. The rolling velocity is obviously the calculated length of tube divided by the observed roll time.

EFFECT ON BALL. An increase in temperature increases the diameter and decreases the density of the ball. The change in ball density is given by the relation

$$\Delta\rho_s = \frac{-3\alpha_b \Delta t}{1 + 3\alpha_b \Delta t} \rho_{so} \qquad (17)$$

The term $(\rho_s - \rho)$ in Equation 16 is mainly affected by the change in fluid density.

EFFECT ON DIAMETER RATIO. If the ball and tube are made of the same material there is no change in the ratio of diameters and no change in the value of the correlation factor, K, when the temperature is increased. A change in the ratio of diameters causes a change in the value of the correlation factor which depends on the value of d/D and the rate of change of K with d/D. This rate is shown graphically in Figure 6. The change in the diameter ratio with temperature is estimated from the individual coefficients of expansion and the relation shown in Equation 18.

$$\Delta \frac{d}{D} = \frac{(\alpha_b - \alpha_t)\ \Delta t}{1 + \alpha_t\ \Delta t}\left(\frac{d}{D}\right)_o \qquad (18)$$

The rate of change of K from Figure 6 is then multiplied by the change in d/D from the equation above to obtain the estimated change in the correlation factor.

EFFECT ON DIAMETER PRODUCT. The linear dimensions of the apparatus combine in the term $d(D + d)$ to affect the calibration when the temperature is changed. A general expression for the change in this term is

$$\Delta[d(D + d)] = [(D_o + 2d_o)\ \alpha_b + D_o\ \alpha_t]\ d_o\, \Delta t \qquad (19)$$

If the ball and tube are made of the same material the coefficient of the calibrating equation still changes because of the change in the value of this diameter product.

Comparison with Data in the Literature

Apparently the work of Hoeppler (*10*) was done on apparatus almost identical with the instrument now marketed under his name. Although he failed to give the diameter of tube and balls, other details, including the ratio of the radius of the tube to radius of ball, were given. In describing the commercial instrument Knop (*12*) gave the diameter as 1.5985 cm. and Schrader (*15*) gave 1.5987 cm. Using the data of Hoeppler on air and castor oil, the values of the resistance factor and Reynolds number were calculated for seven ball sizes. The correlation factor, K, was calculated from the single values for each condition. The detailed calculations are given in Table II and the calculated values of K are plotted on Figure 4.

Four out of six values of K calculated from these data check the present work, and the remaining two values are within about 40 per cent of the corresponding values read from the curve. Hoeppler's instrument was almost twice as large in diameter and inclined at an angle of 80° from the horizontal. Since sliding motion of the balls was probably more pronounced with this instrument, the agreement of these data with the present work is unexpectedly good. The values of Reynolds number limiting the streamline region cannot be applied to this instrument.

Benning and Markwood (*2*) used a modified Hoeppler instrument to measure the viscosity of gases and liquids. They derived a calibration equation for the ball and tube combination used on gases from an interpolated value of roll time in air at 20° C. Measurements in air were made at temperatures from 1.4° to 79.5° C., and the viscosity was calculated from the experimental values. The calculated viscosity did not check the critical values (*11*) for air and the deviation from the critical values was greater than the normal dispersion of data for air. Using the original data and the viscosity of air from International Critical Tables, the correlation factor, K, was calculated at seven temperatures. There was a definite trend of the values with changing temperature.

Apparently the ball used in this work was of glass. If it is assumed that the ball and tube material were the same, the diameter ratio and correlation factor would not change with temperature. The increase in diameter product in the temperature range used was small and did not account for the change indicated. It is concluded that the ball and tube were not made of material having the same coefficient of thermal expansion.

The ball and tube combination used on liquids and vapors was calibrated with water and two other known liquids at temperatures from 0° to 60° C. A definite trend in the calculated correlation factor was not evident, but in this case the diameter ratio was smaller and the apparatus was not so sensitive to temperature changes. The calculated values agree well with themselves but are 40 per cent smaller than the values found in the present work. The detailed calculations on the data of Benning and Markwood are given in Table III and the values of the correlation factor calculated from their data are plotted in Figure 4.

Spée (*16*) described experimental measurements of the fluid velocity required to suspend spheres in inclined tapered glass tubes. The full effective force of gravity was used in maintaining the position of the ball against the flowing fluid. In four cases of streamline flow values of the resistance factor, Reynolds number, and correlation factor were calculated. The detailed calculations are given in Table III. The points plotted on Figure 4 agree with the present work in two out of four cases, but the other values are of the right order of magnitude.

Summary

By the use of dimensional analysis, the variables involved in the calibration of the rolling ball viscometer have been combined in the Reynolds number containing the variable viscosity and a resistance factor proportional to the driving force on the ball. A general equation showing the relation between all variables was obtained from a correlation of these factors.

The usual calibration of the instrument in the streamline region of fluid flow was readily obtained from the general equation. The coefficient of this viscometer calibration can now be predicted from the dimensions of the instrument.

When data are taken with the rolling ball viscometer in the turbulent region of flow, the true viscosity can be estimated by applying an empirical correction to the viscosity calculated from the calibration valid only for the streamline region of flow.

A study of the effect of a change in operating temperature on the calibration has been made for the first time. The effect of temperature is appreciable when the coefficients of expansion are different, when temperature changes are large, and when close-fitting balls are used.

Because of the greater sensitivity to viscosity in the streamline region of fluid flow, best results as a viscometer are obtained when the instrument is used in this region. The relations presented have practical value in allowing the viscometer to be designed for a specific purpose or in allowing its range in the more applicable streamline region to be determined. A study of the variables involved will indicate the best design which should result in experimental measurements of greater reliability.

Literature Cited

(1) Awberry, J. H., and Griffiths, E., *Proc. Roy. Soc. Edinburgh*, **47**, 1–10 (1926).
(2) Benning, A. F., and Markwood, W. H., *Refrig. Eng.*, **37**, 243–7 (1939).
(3) Bingham, E. C., "Fluidity and Plasticity", pp. 295–318, New York, McGraw-Hill Book Co., 1922.
(4) Block, R. B., *J. Applied Phys.*, **11**, 635–42 (1940).
(5) *Ibid.*, **13**, 56–65 (1942).
(6) Bridgman, P. W., "Dimensional Analysis", 1st ed., pp. 65–7, New Haven, Yale University Press, 1922.
(7) Flowers, A. E., *Proc. Am. Soc. Testing Materials*, **14**, II, 565–616 (1914).
(8) Hersey, M. D., *J. Wash. Acad. Sci.*, **6**, 525–30 (1916).
(9) Hersey, M. D., and Shore, H., *Mech. Eng.*, **50**, 221–32 (1928).
(10) Hoeppler, F., *Z. tech. Physik*, **14**, 165–9 (1933).
(11) "International Critical Tables", Vol. 5, p. 2, New York, McGraw-Hill Book Co., 1929.
(12) Knop, W., *Z. Ver. deut. Zucker-Ind.*, **83**, 932–41 (1933).
(13) Sage, B. H., IND. ENG. CHEM., ANAL. ED., **5**, 261–3 (1933).
(14) Sage, B. H., and Lacey, W. N., *Trans. Am. Inst. Mining Met. Engrs.*, **127**, 118–34 (1938).
(15) Schrader, H., *Pharm. Zentralhalle*, **75**, 689–93 (1934).
(16) Spée, V., "Trans. Chem. Engr. Congr. World Power Conf.", Vol. 2, pp. 1–29, London, Percy Lund, Humphries and Co., 1937.